1Q A simple pendulum of length L and mass M has frequency F. To increase its frequency to 2f: decrease its length to L/4

May 13, 2019

Today: CH 15 HW due; Quiz
Wed: lab 11 due
Thurs: PLC #15 due
Next mon: 11-am – 1pm (Review session
Next tues: 10AM – 12:50 (Final EXAM)

CHAPTER 16: Traveling Waves

SHM → $\boxed{x(t) = A\cos(\omega t + \phi_0)}$

ω → angular frequency

$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$

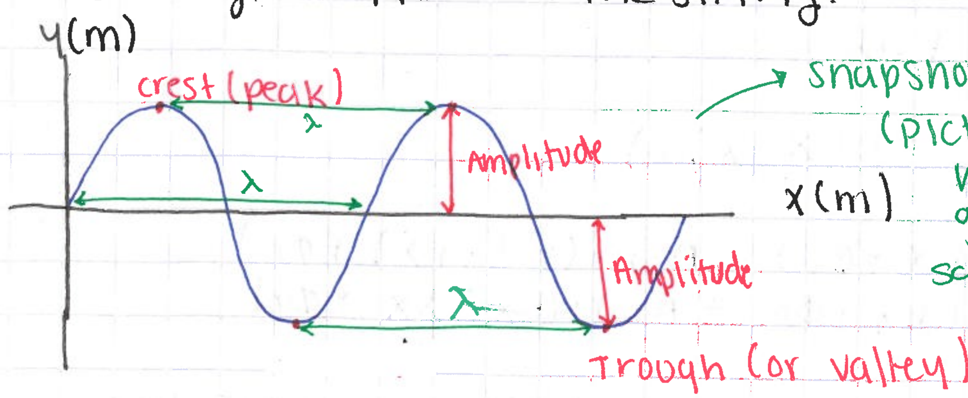
ϕ_0 → Phase angle

$x(t=0s) = A\cos\phi_0$

$v(t=0s) = -A\omega\sin\phi_0$

} Describes initial conditions of system.

1Q: A sinusoidal transverse wave is traveling on a string. Any point on the string:



→ snapshot graph (picture of the wave at a given time - say t = 0)

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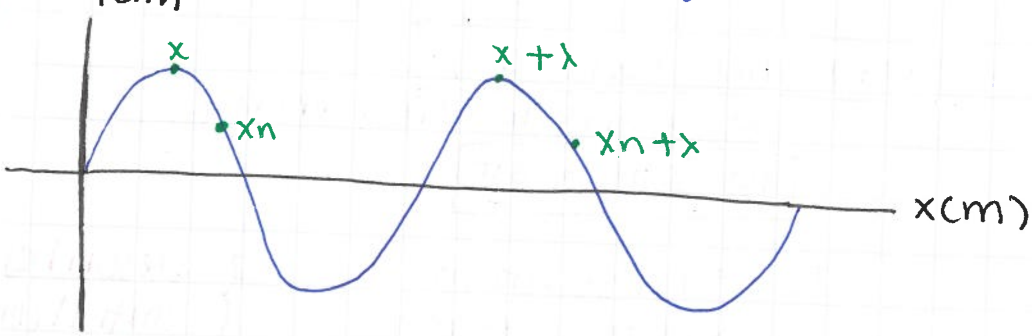
To completely describe a wave (on a string), w need a function that gives the shape of the wave as a function of position & time

For a wave traveling in the +x direction)

$$\boxed{D(x,t) = A\sin(kx - \omega t + \phi_0)}$$

Displacement as a function of x and t

$A \rightarrow$ amplitude in meters
$\phi_0 \rightarrow$ Phase constant
$\omega \rightarrow$ Angular frequency in rad/s
$k \rightarrow$ Angular wave number in rad/m or m^{-1}



* We must have $D(x,t) = D(x+\lambda, t)$

Displacement must be the same when wave travels one wavelength

choose t=0s

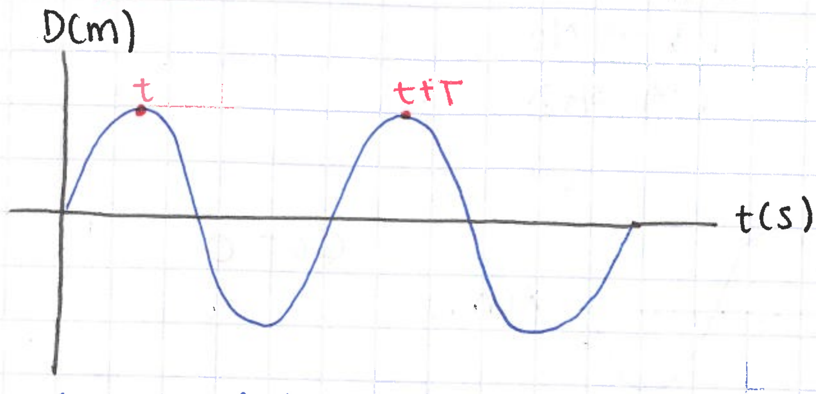
$D(x,0) = D(x-\lambda, 0)$

$A\sin(kx + \phi_0) = A\sin(k(x+\lambda) + \phi_0)$
$A\sin(kx + \phi_0) = A\sin(kx + k\lambda + \phi_0)$

* Since sin repeats itself every 2π we must have $k\lambda = 2\pi$

$K\lambda = 2\pi$

$$\boxed{K = 2\pi/\lambda}$$



History graph

Graph of displacement versus time at a certain point along the string (call it $x=0$)

Since Displacement must be the same when time increases by one period:

$$D(0,t) = D(0,t+T)$$

$$A\sin(-\omega t + \phi) = A\sin(-\omega(t+T)+\phi_0)$$

$$A\sin(-\omega t + \phi_0) = A\sin(-\omega t - \omega T + \phi_0)$$

$$\sin(-\phi) = -\sin\phi$$

$$-A\sin(\omega t - \phi_0) = -A\sin(\omega t + \omega T - \phi_0)$$

* Since sin repeats itself every 2π, ω must have $\omega T = 2\pi$

$\omega T = 2\pi$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$ → angular frequency
$[\omega] = rad/s$

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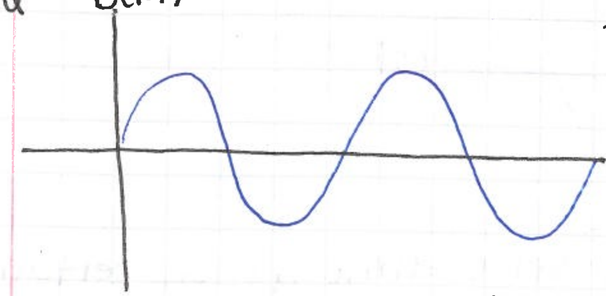
Conceptual Question 16.A

$1-5 \rightarrow 2\pi$	$1-5 \rightarrow \lambda$
$1-3 \rightarrow \pi$	$1-2 \rightarrow \lambda/4$
$1-2 \rightarrow \pi/2$	$1-3 \rightarrow \lambda/2$
$1-4 \rightarrow 3\pi/2$	$1-4$ $3/2\,\lambda$

1Q



$\phi_0 = 0$

$D(x,t) = A\sin(kx - \omega t + \phi_0$

at $t = 0s$, $x = 0m$, $D(0,0) = 0$

$D(0,0) = A\sin\phi_0 = 0$

$\sin\phi_0 = 0$ $\phi_0 = 0$

Recap

SHM $\rightarrow$ $\boxed{x(t) = A\cos(\omega t + \phi_0)}$

$\boxed{K = 2\pi/\lambda}$

$\boxed{\omega = 2\pi f = 2\pi/T}$

A → amplitude in m

K → angular wave number in m^{-1} or rad/m

ω → Angular frequency in s^{-1} or rad/s

ϕ_0 → Phase constant

↳ describes initial conditions

For a wave traveling in the +x-direction

$\boxed{D(x,t) = A\sin(Kx - \omega t + \phi_0)}$

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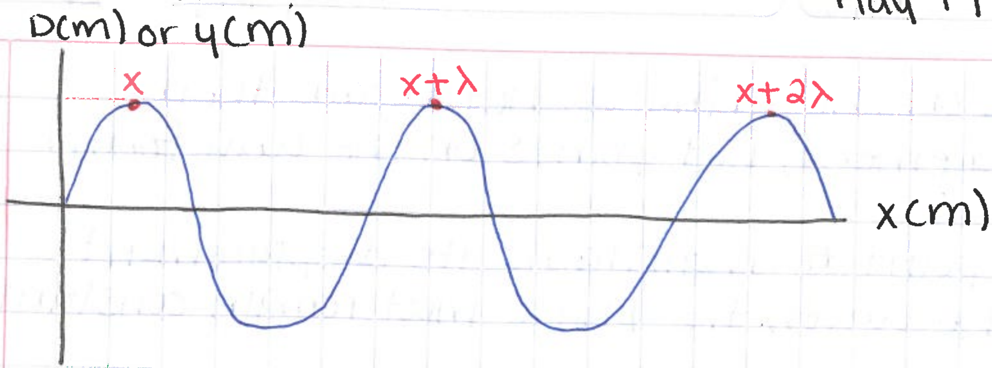
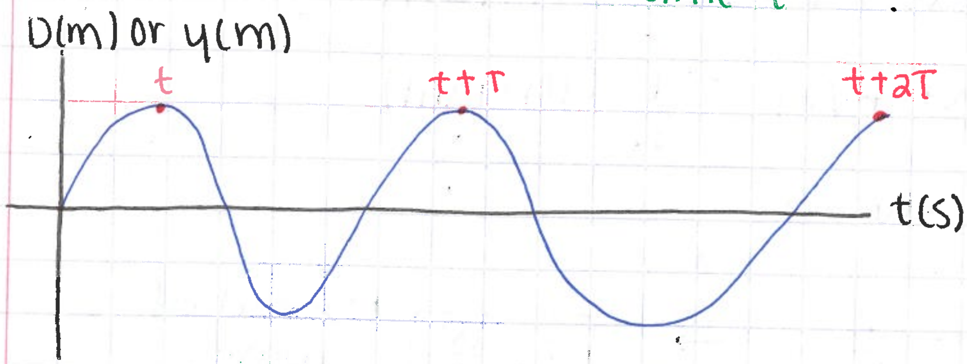
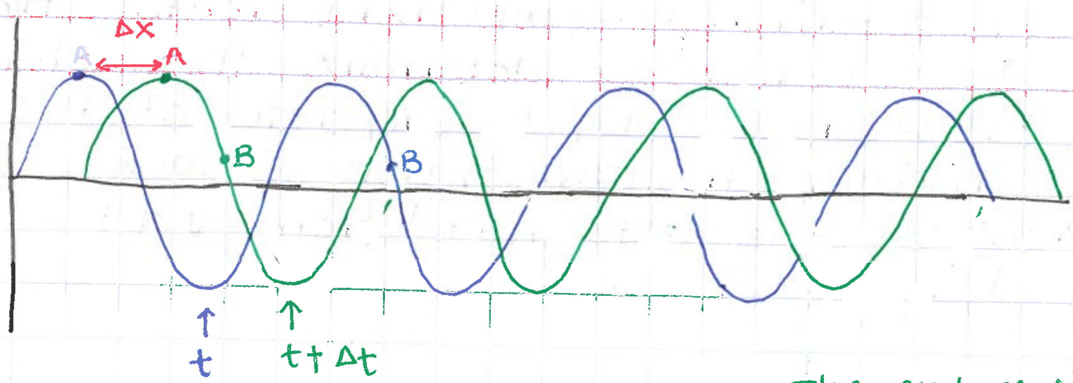


SNAPSHOT GRAPH: Picture of wave at a given time t.



HISTORY GRAPH: Graph of displacement versus time for a certain point along the string

Wave Speed



A → Point on the wave form

The entire wave pattern moves a distance Δx in a time Δt.

$$\boxed{\text{Wavespeed} \quad v = \frac{\Delta x}{\Delta t}}$$

* Points on the string do not retain their displacement, but points on the wave form do
  - If point A maintains its displacement as the waves, the phase must remain constant.

$D(x,t) = A\sin(kx - \omega t + \phi_0)$

$kx - \omega t + \phi_0 = \text{constant}$

$$\frac{d}{dt}(kx - \omega t + \phi_0) = \frac{d}{dx}(\text{constant})$$

$$\frac{k\,dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} \longrightarrow \boxed{v = \frac{\omega}{k}}$$ → Speed of wave

- $2\pi f \quad k = 2\pi/\lambda \quad v = \omega/k = \dfrac{2\pi f}{(2\pi/\lambda)} = \lambda f$

$\boxed{v = \lambda f}$ → True for every kind of wave

- f → is determined by whether is creating the waves (which also determines the amplitude)
- v → is determined by the properties of the medium the wave travels through
- $\lambda = \dfrac{v}{f}$ → λ is determined from v & f

- Speed of Light in a material is given by:

$$\boxed{v = c/n}$$

$v \rightarrow$ speed of light in a material
$c \rightarrow$ speed of light in a vacuum
($c = 2.99 \times 10^8$ m/s)
$n \rightarrow$ index of refraction
$n_{water} \simeq 1.33$
$n_{glass} \simeq 1.50$
$n_{air} \simeq 1.0$

- Speed of a wave on a stretched string

In general:

$$v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$$

$$\boxed{v = \sqrt{\frac{T_s}{\mu}}} \rightarrow \text{speed of waves on a string}$$

$$\boxed{\mu = \frac{m}{L}}$$

- $T_s \rightarrow$ Tension in the string in N
- $\mu \rightarrow$ linear mass density in kg/m

Problem 16.12

$$D(x,t) = (3.5\text{cm})\sin\left[\underbrace{(2.7\tfrac{rad}{m})x}_{kx} \underbrace{-(124\tfrac{rad}{s})t}_{-\omega t} \underbrace{+0}_{+\phi_0}\right]$$

$$\Rightarrow k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} \rightarrow \frac{2\pi}{(2.7\frac{rad}{m})} = 2.33\text{m}$$

$$\rightarrow \omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi} = \frac{124\frac{rad}{s}}{2\pi} = 19.7\text{Hz}$$

$$v = \omega/k = \lambda f \quad \rightarrow v = \lambda f = (2.33\text{m})(19.7\text{Hz})$$

$$v = 46\frac{m}{s}$$

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Problem 16.59

$\lambda f = v$

$\lambda = v/f$

$v = \sqrt{\frac{T_S}{\mu}}$

$v_1 = \sqrt{\frac{(2250N)}{(0.009 kg/m)}} = 500 m/s$

$v_2 = \sqrt{\frac{2250N}{0.025 kg/m}} = 300 m/s$

$\lambda_1 = \frac{v_1}{f_1} = \frac{500 m/s}{1500 Hz} = \frac{1}{3} m$

$\lambda_2 = \frac{v_2}{f_2} = \frac{300 m/s}{1500 Hz} = \frac{1}{5} m$

# of wavelength per section

$1 \rightarrow L_1/\lambda_1 = \frac{1.0m}{(\frac{1}{3}m)} = 3$

$2 \rightarrow L_2/\lambda_2 = \frac{1.0m}{(\frac{1}{5}m)} = 5$

Total number of complete wave $= \boxed{8}$